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Corrigendum for Vector Equilibrium Problems. Existence Theorems and Convexity of Solution Set

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In the proof of $E \subset \bigcap_{f\in C^\sharp} T(f)$ of Theorem 3 of [1], there is a gap. Theorem 3 of [1] should reads as follows.

THEOREM 3. *Let X, Y, D and C be as in Theorem 1, and let $C^\sharp \neq \emptyset$. Let $G,\ H: D\times D \longrightarrow 2^Y$ be set-valued mappings satisfying the conditions (i)–(vi) in Theorem 1. In addition, assume that G, H satisfy the following condition:*

(vii) for any fixed $y \in D$, $G(., y) + H(., y): D \longrightarrow 2^Y$ is proper quasi-C-concave, i.e., for any $x_1, x_2 \in D$, $t \in [0,1]$, $x = tx_1 + (1-t)x_2$, and for any $w \in G(x,y) + H(x,y)$, there exists $w_1 \in G(x_1, y) + H(x_1, y)$ or $w_2 \in G(x_2, y) + H(x_2, y)$ such that $w \in w_1 + C$ or $w \in w_2 + C$.

Then the solution set of (VEP 1)

$$E = \{\bar{x} \in K: G(\bar{x}, y) + H(\bar{x}, y) \subset Y\backslash(-\mathrm{int}C), \quad \forall y \in D\}$$

is convex. If C satisfies the condition (Δ), then the solution set of (VEP 3)

$$E_* = \{\bar{x} \in K: G(\bar{x}, y) + H(\bar{x}, y) \subset Y\backslash(-C\backslash\{0\}), \quad \forall y \in D\}$$

is convex.

Proof. We only show the case of (VEP 1). The other one can be proven similarly. By Theorem 1, $E \neq \emptyset$. Since C is a closed, convex pointed cone with $\mathrm{int}\, C \neq \emptyset$, we have

$$Y\backslash(-\mathrm{int}C) + C = Y\backslash(-\mathrm{int}C), \tag{1}$$

and

$$Y\backslash(-C\backslash\{0\}) + C = Y\backslash(-C\backslash\{0\}). \tag{2}$$

Let $x_1, x_2 \in E$, $t \in [0,1]$ and $x = tx_1 + (1-t)x_2$. We show that $x \in E$. It follows from $x_i \in E$, $(i = 1, 2)$ that

$$G(x_i, y)+H(x_i, y)\subset Y\backslash(-\mathrm{int}C), \quad \forall y\in D, \; i=1,2. \tag{3}$$

For any $w \in G(x, y) + H(x, y)$, by the condition (vii), we have $w_1 \in G(x_1, y)+H(x_1, y)$ or $w_2\in G(x_2, y)+H(x_2, y)$ such that

$$w\in w_1+C \quad \text{or} \quad w\in w_2+C. \tag{4}$$

By (1),(3) and (4), we have $w\in Y\backslash(-\mathrm{int}C)$. Since w is arbitrary, we get $x\in E$. □

Theorem 4 of [1] should reads as follows.

THEOREM 4. *Let X,Y,D and C be the same as in Theorem 3. Let G, $H: D\times D \longrightarrow 2^Y$ satisfy all the conditions in Theorem 2. In addition, assume that the following condition holds:*

(vi) for any fixed $y\in D$, $G(.,y)+H(.,y) : D\longrightarrow 2^Y$ is proper quasi-C-concave. Then the solution set of (VEP 1)

$$E=\{\bar{x}\in B: G(\bar{x}, y)+H(\bar{x}, y)\subset Y\backslash(-\mathrm{int}C), \quad \forall y\in D\}$$

is convex. If C satisfies condition (Δ)*, then the solution set of (VEP 3)*

$$E_*=\{x\bar{\in}B: G(\bar{x}, y)+H(\bar{x}, y)\subset Y\backslash(-C\backslash\{0\}), \quad \forall y\in D\}$$

is convex.

Reference

1. Fu, J.Y. (2005), Vector equilibrium problems. Existence theorem and convexity of solution set, *Journal of Global Optimization* 31, 109–119.